

# TABLE OF LEARNING MATERIALS I-1 MATHEMATICS (6A-2A) / PENCIL SKILLS PROGRAMME (ZI-ZII)

KUMON

The black mark (■) indicates Starting Points.  
SCT: Standard Completion Time (Min./Sheet)  
This is the time in which the student should complete each worksheet, including time taken for corrections.  
(SCT is not used in Levels 6A-5A and ZI-ZII.)

December 2012

Pencil Skills Programme		6A		5A		4A		SCT	3A		SCT	2A		SCT
1- 10	Colouring 1	1- 10	Counting (Up to 5) 1	Number Reading Exercises (Up to 30) 1	Number Tracing Exercises 1	0.5-2	Numbers up to 100 Part 1	0.5-2	Numbers up to 100 Part 1	0.5-2	Review up to 3A	1-2		
11- 20	Colouring 2	11- 20	Counting (Up to 5) 2	Number Reading Exercises (Up to 30) 2	Number Tracing Exercises 2	0.5-2	Numbers up to 100 Part 2	0.5-2	Adding 4 Part 1 (Up to 12+4)	1-2				
21- 30	Straight Lines 1	21- 30	Counting (Up to 5) 3	Number Reading Exercises (Up to 30) 3	Number Tracing Exercises 3	0.5-2	Numbers up to 100 Part 3	0.5-2	Adding 4 Part 2 (Up to 16+4)	1-2				
31- 40	Straight Lines 2	31- 40	Counting (Up to 10) 1	Number Reading Exercises (Up to 30) 4	Number Tracing Exercises 4	0.5-2	Numbers up to 100 Part 4	0.5-2	Adding 5 Part 1 (Up to 12+5)	1-2				
41- 50	Straight Lines 3	41- 50	Counting (Up to 10) 2	Number Reading Exercises (Up to 30) 5	Number Writing Exercises up to 10 Part 1	0.5-2	Numbers up to 100 Part 5	0.5-2	Adding 5 Part 2 (Up to 15+5)	1-2				
51- 60	Curved Lines 1	51- 60	Counting (Up to 10) 3	Number Reading Exercises (Up to 30) 6	Number Writing Exercises up to 10 Part 2	0.5-2	Numbers up to 100 Part 6	0.5-2	Adding up to 5 Part 1	1-2				
61- 70	Curved Lines 2	61- 70	Counting (Up to 10) 4	Number Reading Exercises (Up to 30) 7	Number Writing Exercises up to 10 Part 3	0.5-2	Numbers up to 120	1-2	Adding up to 5 Part 2	1-2				
71- 80	Curved Lines 3	71- 80	Counting (Up to 10) 5	Number Reading Exercises (Up to 30) 8	Number Writing Exercises up to 10 Part 4	0.5-2	Adding 1 Part 1 (Up to 12+1)	1-2	Adding 6 Part 1 (Up to 12+6)	1-2				
81- 90	Curved Lines 4	81- 90	Counting (Up to 10) 6	Number Reading Exercises (Up to 30) 9	Number Writing Exercises up to 10 Part 5	0.5-2	Adding 1 Part 2 (Up to 18+1)	1-2	Adding 6 Part 2 (Up to 14+6)	1-2				
91-100	Curved Lines 5	91-100	Counting (Up to 10) 7	Number Reading Exercises (Up to 30) 10	Number Writing Exercises up to 10 Part 6	0.5-2	Adding 1 Part 3 (Up to 24+1)	1-2	Adding 7 Part 1 (Up to 11+7)	1-2				
1- 10	Shapes and Pictures 1	101-110	Number Reading Exercises (Up to 10) 1	Sequence of Numbers (Up to 30) 1	Number Writing Exercises up to 20 Part 1	0.5-2	Adding 1 Part 4 (Up to 30+1)	1-2	Adding 7 Part 2 (Up to 13+7)	1-2				
11- 20	Shapes and Pictures 2	111-120	Number Reading Exercises (Up to 10) 2	Sequence of Numbers (Up to 30) 2	Number Writing Exercises up to 20 Part 2	0.5-2	Adding 1 Part 5 (Up to 60+1)	1-2	Adding up to 7 Part 1	1-2				
21- 30	Shapes and Pictures 3	121-130	Number Reading Exercises (Up to 10) 3	Sequence of Numbers (Up to 30) 3	Number Writing Exercises up to 30 Part 1	0.5-2	Adding 1 Part 6 (Up to 1000+1)	1-2	Adding up to 7 Part 2	1-2				
31- 40	Shapes and Pictures (Stories) 1	131-140	Number Reading Exercises (Up to 10) 4	Sequence of Numbers (Up to 40) 1	Number Writing Exercises up to 30 Part 2	0.5-2	Adding 2 Part 1 (Up to 14+2)	1-2	Adding 8 Part 1 (Up to 11+8)	1-2				
41- 50	Shapes and Pictures (Stories) 2	141-150	Number Reading Exercises (Up to 10) 5	Sequence of Numbers (Up to 40) 2	Numbers up to 50 Part 1	0.5-2	Adding 2 Part 2 (Up to 18+2)	1-2	Adding 8 Part 2 (Up to 12+8)	1-2				
51- 60	Shapes and Pictures (Stories) 3	151-160	Number of Dots (Up to 10) 1	Sequence of Numbers (Up to 40) 3	Numbers up to 50 Part 2	0.5-2	Adding 2 Part 3 (Up to 32+2)	1-2	Adding 9 (Up to 12+9)	1-2				
61- 70	Back and Forth 1	161-170	Number of Dots (Up to 10) 2	Sequence of Numbers (Up to 50) 1	Numbers up to 50 Part 3	0.5-2	Adding 3 Part 1 (Up to 14+3)	1-2	Adding 9 and 10 (Up to 12+9 and 15+10)	1-2				
71- 80	Back and Forth 2	171-180	Number of Dots (Up to 10) 3	Sequence of Numbers (Up to 50) 2	Numbers up to 50 Part 4	0.5-2	Adding 3 Part 2 (Up to 21+3)	1-2	Adding up to 10 Part 1	1-2				
81- 90	Corners and Curves 1	181-190	Number of Dots (Up to 10) 4	Sequence of Numbers (Up to 50) 3	Numbers up to 50 Part 5	0.5-2	Adding up to 3 Part 1	1-2	Adding up to 10 Part 2	1-2				
91-100	Corners and Curves 2	191-200	Number of Dots (Up to 10) 5	Large Numbers	Numbers up to 50 Part 6	0.5-2	Adding up to 3 Part 2	1-2	Adding up to 10 Part 3	1-2				

## Levels ZI & ZII (Shapes and Pictures)



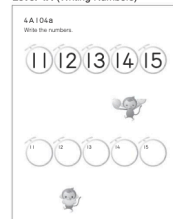
## Level 6A (Counting)



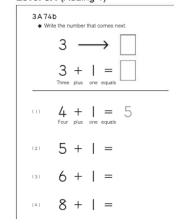
## Level 5A (Reading Numbers)



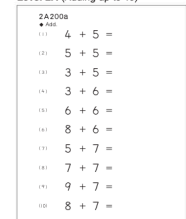
## Level 4A (Writing Numbers)



## Level 3A (Adding 1)



## Level 2A (Adding up to 10)



# TABLE OF LEARNING MATERIALS I-2 MATHEMATICS (A-F)

KUMON

The black mark (■) indicates Starting Points.  
SCT: Standard Completion Time (Min./Sheet)  
This is the time in which the student should complete each worksheet, including time taken for corrections.

October 2016

	A	SCT	B	SCT	C	SCT	D	SCT	E	SCT	F	SCT
1- 10	Addition 1 (Review up to 2A)	1-2	Addition (Review up to A)	1-2	Review up to B	2-3	Review up to C	2-3	Review up to D	2-3	Review up to E 1	3-5
11- 20	Addition 2 (Up to sum of 12)	1-2	Addition to 100 Part 1	1-2	Multiplication up to 3	2-3	Multiplication: 2 Digits X 2 Digits 1	2-4	Addition of Fractions 1	3-3	Review up to E 2	3-5
21- 30	Addition 3 (Up to sum of 15)	1-2	Addition to 100 Part 2	2-3	Multiplication up to 5	2-3	Multiplication: 2 Digits X 2 Digits 2	2-4	Addition of Fractions 2	3-4	Multiplication and Division of 3 Fractions	4-6
31- 40	Addition 4 (Up to sum of 18)	1-2	Addition to 100 Part 3	2-3	Multiplication up to 7	2-3	Multiplication: 2 Digits X 2 Digits 3	3-4	Addition of Fractions 3	3-4	Addition of 3 Fractions 1	3-5
41- 50	Addition 5 (Up to sum of 20)	1-2	Addition of 2-Digit Numbers 1	2-3	Multiplication up to 9	2-3	Multiplication: 3 Digits X 2 Digits	3-5	Addition of Fractions 4	3-4	Addition of 3 Fractions 2	4-6
51- 60	Addition 6 (Up to sum of 24)	1-2	Addition of 2-Digit Numbers 2	2-3	Multiplication: 2 Digits X 1 Digit 1	2-4	Addition and Subtraction	2-4	Addition of Fractions 5	3-5	Addition and Subtraction of 3 Fractions	4-6
61- 70	Addition 7 (Up to sum of 28)	1-2	Addition of 2-Digit Numbers 3	2-3	Multiplication: 2 Digits X 1 Digit 2	2-4	Multiplication and Division 1	3-4	Addition of Fractions 6	3-5	Order of Operations (3 Fractions) 1	3-5
71- 80	Addition 8 (Summary of addition)	2-3	Addition of 3-Digit Numbers 1	2-4	Multiplication: 2 Digits X 1 Digit 3	2-4	Multiplication and Division 2	3-4	Addition of Fractions 7	3-5	Order of Operations (3 Fractions) 2	3-5
81- 90	Subtraction 1 (Subtracting 1)	1-2	Addition of 3-Digit Numbers 2	2-4	Multiplication: 2 Digits X 1 Digit 4	2-4	Division by 2-Digit Numbers 1	3-4	Addition of Fractions 8	3-5	Order of Operations (3 Fractions) 3	3-5
91-100	Subtraction 2 (Subtracting 2)	1-2	Addition of 3-Digit Numbers 3	3-5	Multiplication: 2 Digits X 1 Digit 5	2-4	Division by 2-Digit Numbers 2	3-5	Addition of Fractions 9	4-6	Order of Operations (3-or-More Fractions) 1	4-6
101-110	Subtraction 3 (Subtracting 3)	1-2	Subtraction 1 (Review up to A)	1-2	Multiplication: 3 or 4 Digits X 1 Digit	3-5	Division by 2-Digit Numbers 3	3-5	Subtraction of Fractions 1	3-4	Order of Operations (3-or-More Fractions) 2	4-6
111-120	Subtraction 4 (Subtracting up to 3)	1-2	Subtraction 2 (Review up to A)	2-3	Introduction to Division	2-3	Division by 2-Digit Numbers 4	3-5	Subtraction of Fractions 2	3-5	Order of Operations (3-or-More Fractions) 3	4-6
121-130	Subtraction 5 (Subtracting up to 5)	1-2	Subtraction of 2-Digit Numbers 1	2-3	Division with Remainders 1	2-3	Division by 2-Digit Numbers 5	3-5	Subtraction of Fractions 3	3-5	Order of Operations (3-or-More Fractions) 4	4-6
131-140	Subtraction 6 (From numbers up to 10)	1-2	Subtraction of 2-Digit Numbers 2	2-3	Division with Remainders 2	2-3	Division: Quotients of 2-or-More Digits 1	3-5	Addition and Subtraction of Fractions	4-6	Fractions and Decimals 1	3-5
141-150	Subtraction 7 (From numbers up to 11)	1-2	Subtraction of 2-Digit Numbers 3	2-3	Division with Remainders 3	2-3	Division: Quotients of 2-or-More Digits 2	4-6	Multiplication of Fractions 1	3-4	Fractions and Decimals 2	4-6
151-160	Subtraction 8 (From numbers up to 12)	1-2	Addition and Subtraction of 2-Digit Numbers	2-4	Division with Remainders 4	2-3	Fractions	3-5	Multiplication of Fractions 2	3-5	Fractions and Decimals 3	4-6
161-170	Subtraction 9 (From numbers up to 14)	1-2	Subtraction of 3-Digit Numbers 1	2-3	Division: 2 Digits ÷ 1 Digit 1	2-3	Reduction 1	2-3	Division of Fractions	3-5	Solving Word Problems 1	4-6
171-180	Subtraction 10 (From numbers up to 16)	1-2	Subtraction of 3-Digit Numbers 2	2-4	Division: 2 Digits ÷ 1 Digit 2	2-3	Reduction 2	2-3	Multiplication and Division of Fractions	3-5	Solving Word Problems 2	5-7
181-190	Subtraction 11 (From numbers up to 20)	1-2	Subtraction of 3-Digit Numbers 3	2-4	Division: 3 Digits ÷ 1 Digit 1	3-4	Reduction 3	2-4	Four Operations of 2 Fractions 1	3-5	Decimals 1	3-5
191-200	Subtraction 12 (Summary of subtraction)	2-3	Subtraction of 3-Digit Numbers 4	3-5	Division: 3 Digits ÷ 1 Digit 2	3-5	Reduction 4	2-4	Four Operations of 2 Fractions 2	3-5	Decimals 2	3-5

## Level A (Subtraction)

**A200a**  
• Sub  
(1)  $10 - 3 =$   
(2)  $11 - 6 =$   
(3)  $12 - 7 =$   
(4)  $15 - 4 =$   
(5)  $16 - 8 =$   
(6)  $13 - 7 =$   
(7)  $17 - 9 =$   
(8)  $14 - 5 =$   
(9)  $18 - 6 =$   
(10)  $19 - 2 =$   
(11)  $20 - 3 =$

## Level B (Addition to 100)

**B21a**  
• Add  
(1)  $\begin{array}{r} 15 \\ + 5 \\ \hline \end{array}$  (12)  $\begin{array}{r} 15 \\ + 15 \\ \hline \end{array}$   
(2)  $\begin{array}{r} 25 \\ + 5 \\ \hline \end{array}$  (13)  $\begin{array}{r} 25 \\ + 15 \\ \hline \end{array}$   
(3)  $\begin{array}{r} 35 \\ + 5 \\ \hline \end{array}$  (14)  $\begin{array}{r} 35 \\ + 15 \\ \hline \end{array}$   
(4)  $\begin{array}{r} 45 \\ + 5 \\ \hline \end{array}$  (15)  $\begin{array}{r} 45 \\ + 15 \\ \hline \end{array}$   
(5)  $\begin{array}{r} 55 \\ + 5 \\ \hline \end{array}$  (16)  $\begin{array}{r} 55 \\ + 15 \\ \hline \end{array}$

## Level C (Multiplication)

**C32a**  
• Mult  
(1)  $\begin{array}{r} 32 \\ \times 2 \\ \hline \end{array}$  (12)  $\begin{array}{r} 53 \\ \times 3 \\ \hline \end{array}$   
(2)  $\begin{array}{r} 42 \\ \times 2 \\ \hline \end{array}$  (13)  $\begin{array}{r} 63 \\ \times 3 \\ \hline \end{array}$   
(3)  $\begin{array}{r} 52 \\ \times 2 \\ \hline \end{array}$  (14)  $\begin{array}{r} 73 \\ \times 3 \\ \hline \end{array}$   
(4)  $\begin{array}{r} 62 \\ \times 2 \\ \hline \end{array}$  (15)  $\begin{array}{r} 83 \\ \times 3 \\ \hline \end{array}$   
(5)  $\begin{array}{r} 72 \\ \times 2 \\ \hline \end{array}$  (16)  $\begin{array}{r} 93 \\ \times 3 \\ \hline \end{array}$

## Level D (Division)

**D31a**  
• Div  
(1)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$  (12)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$   
(2)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$  (13)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$   
(3)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$  (14)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$   
(4)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$  (15)  $\begin{array}{r} \square \\ 2 \overline{) 15} \end{array}$

## Level E (Addition of Fractions)

**E70a**  
• Add  
(1)  $\frac{1}{2} + \frac{1}{2} =$   
(2)  $\frac{1}{2} + \frac{1}{3} =$   
(3)  $\frac{1}{3} + \frac{1}{3} =$   
(4)  $\frac{1}{3} + \frac{1}{6} =$   
(5)  $\frac{1}{6} + \frac{1}{6} =$   
(6)  $\frac{1}{6} + \frac{1}{6} =$

## Level F (Order of Operations)

**F121a**  
• Order  
(1)  $(\frac{1}{2} + \frac{1}{3}) \times 4$  (12)  $\frac{1}{2} \times \frac{1}{2} + 1 \div \frac{1}{2}$   
(2)  $\frac{1}{2} \times \frac{1}{2} + 1 \div \frac{1}{2}$   
(3)  $\frac{1}{2} \times \frac{1}{2} + 1 \div \frac{1}{2}$   
(4)  $\frac{1}{2} \times \frac{1}{2} + 1 \div \frac{1}{2}$   
(5)  $\frac{1}{2} \times \frac{1}{2} + 1 \div \frac{1}{2}$   
(6)  $\frac{1}{2} \times \frac{1}{2} + 1 \div \frac{1}{2}$

# TABLE OF LEARNING MATERIALS II-1 (Level G to L)

KUMON

The black mark (■) indicates most suitable Starting Points.  
SCT: Standard Completion Time (Min/Sheet)  
This is the set time in which the student should complete each worksheet, including time taken for correction.

G		SCT	H		SCT	I		SCT	J		SCT	K		SCT	L		SCT
1- 10	Review up to F 1	3-5	Review of G 1	4-6	Review up to H 1	4-6	Expansion of Polynomial Products	5-8	Review of Linear Functions	4-6	Logarithmic Functions	8-12					
11- 20	Review up to F 2	3-5	Review of G 2	4-6	Review up to H 2	5-7	Factorisation I	5-8	Review of Quadratic Functions	5-8	Graphs of Logarithmic Functions	7-14					
21- 30	Addition and Subtraction of Positive and Negative Numbers 1	2-4	Simultaneous Equations with Two Variables 1	5-8	Square Roots 1	5-7	Factorisation II	5-8	Quadratic Functions and Graphs	6-12	Logarithmic Equations and Inequalities	8-16					
31- 40	Addition and Subtraction of Positive and Negative Numbers 2	3-5	Simultaneous Equations with Two Variables 2	5-8	Square Roots 2	5-7	Factorisation III	6-10	Determining Equations of Quadratic Functions	7-14	Modulus Functions	8-16					
41- 50	Addition and Subtraction of Positive and Negative Numbers 3	3-5	Simultaneous Equations with Two Variables 3	5-8	Square Roots 3	5-7	Factorisation IV	6-10	Maxima and Minima of Quadratic Functions I	7-14	Limits and Derivatives	8-16					
51- 60	Addition and Subtraction of Positive and Negative Numbers 4	4-6	Simultaneous Equations with Two Variables 4	5-8	Quadratic Equations 1	5-7	Factorisation V	7-12	Maxima and Minima of Quadratic Functions II	7-14	Tangents	12-24					
61- 70	Multiplication of Positive and Negative Numbers	3-5	Simultaneous Equations with Two Variables 5	5-8	Quadratic Equations 2	5-7	Fractional Expressions	6-10	Maxima and Minima of Quadratic Functions III	8-16	Relative Maxima and Minima I	15-30					
71- 80	Division of Positive and Negative Numbers	4-6	Simultaneous Equations with Three and Four Variables 1	6-10	Quadratic Equations 3	5-7	Irrational Numbers I	5-8	Quadratic Functions and Equations	7-14	Relative Maxima and Minima II	15-30					
81- 90	Four Operations with Positive and Negative Numbers 1	4-6	Simultaneous Equations with Three and Four Variables 2	6-10	Inequalities 1	4-6	Irrational Numbers II	6-10	Quadratic Functions and Inequalities	7-14	Maxima and Minima I	15-30					
91-100	Four Operations with Positive and Negative Numbers 2	4-6	Application of Equations 1	6-8	Inequalities 2	5-7	Quadratic Equations I	5-8	Quadratic Functions and Solutions of Quadratic Equations	8-16	Maxima and Minima II	15-30					
101-110	Values of Algebraic Expressions 1	4-6	Application of Equations 2	6-8	Linear Functions and Graphs 1	5-7	Quadratic Equations II	6-10	Higher Degree Functions	6-12	Applications to Equations and Inequalities	15-30					
111-120	Values of Algebraic Expressions 2	4-6	Simplifying Monomials and Polynomials 1	5-7	Linear Functions and Graphs 2	5-7	Quadratic Equations and Complex Numbers	6-10	Higher Degree Equations and Inequalities	7-14	Indefinite and Definite Integrals	8-16					
121-130	Simplifying Algebraic Expressions 1	3-5	Simplifying Monomials and Polynomials 2	5-7	Linear Functions and Graphs 3	5-7	Discriminant	6-10	Graphs of Fractional Functions I	7-14	Definite Integrals I	12-24					
131-140	Simplifying Algebraic Expressions 2	3-5	Multiplication of Polynomials	5-7	Linear Functions and Graphs 4	5-7	Root-Coefficient Relationships	6-10	Graphs of Fractional Functions II	7-14	Definite Integrals II	12-24					
141-150	Simplifying Algebraic Expressions 3	4-6	Multiplication Using Formulas	5-7	Quadratic Functions and Graphs 1	6-8	Simultaneous Equations	6-10	Fractional Equations and Inequalities	8-16	Areas I	15-30					
151-160	Simplifying Algebraic Expressions 4	4-6	Factorisation 1	4-6	Quadratic Functions and Graphs 2	6-8	Dividing Polynomials	6-10	Graphs of Irrational Functions	7-14	Areas II	15-30					
161-170	Linear Equations 1	3-5	Factorisation 2	5-7	Quadratic Functions and Graphs 3	6-8	Remainder Theorem	6-10	Irrational Equations and Inequalities	8-16	Volumes	15-30					
171-180	Linear Equations 2	4-6	Factorisation 3	5-7	The Pythagorean Theorem 1	7-10	Factor Theorem	6-10	Exponential Functions	6-12	Velocity and Distance	15-30					
181-190	Linear Equations 3	4-6	Factorisation 4	5-7	The Pythagorean Theorem 2	7-10	Proof of Identities	6-10	Graphs of Exponential Functions	7-14	Summary of Differentiation and Integration I	30-40					
191-200	Linear Equations 4	4-6	Factorisation 5	5-7	The Pythagorean Theorem 3	7-10	Proof of Inequalities	7-12	Exponential Equations and Inequalities	8-16	Summary of Differentiation and Integration II	30-40					

## Level G (Simplifying Algebraic Expressions)

G145a

■ Simplify

1. Simplify  $2a + 3b + 4c + 5d + 6e + 7f + 8g + 9h + 10i + 11j + 12k + 13l + 14m + 15n + 16o + 17p + 18q + 19r + 20s + 21t + 22u + 23v + 24w + 25x + 26y + 27z + 28aa + 29bb + 30cc + 31dd + 32ee + 33ff + 34gg + 35hh + 36ii + 37jj + 38kk + 39ll + 40mm + 41nn + 42oo + 43pp + 44qq + 45rr + 46ss + 47tt + 48uu + 49vv + 50ww + 51xx + 52yy + 53zz + 54aa + 55bb + 56cc + 57dd + 58ee + 59ff + 60gg + 61hh + 62ii + 63jj + 64kk + 65ll + 66mm + 67nn + 68oo + 69pp + 70qq + 71rr + 72ss + 73tt + 74uu + 75vv + 76ww + 77xx + 78yy + 79zz + 80aa + 81bb + 82cc + 83dd + 84ee + 85ff + 86gg + 87hh + 88ii + 89jj + 90kk + 91ll + 92mm + 93nn + 94oo + 95pp + 96qq + 97rr + 98ss + 99tt + 100uu + 101vv + 102ww + 103xx + 104yy + 105zz + 106aa + 107bb + 108cc + 109dd + 110ee + 111ff + 112gg + 113hh + 114ii + 115jj + 116kk + 117ll + 118mm + 119nn + 120oo + 121pp + 122qq + 123rr + 124ss + 125tt + 126uu + 127vv + 128ww + 129xx + 130yy + 131zz + 132aa + 133bb + 134cc + 135dd + 136ee + 137ff + 138gg + 139hh + 140ii + 141jj + 142kk + 143ll + 144mm + 145nn + 146oo + 147pp + 148qq + 149rr + 150ss + 151tt + 152uu + 153vv + 154ww + 155xx + 156yy + 157zz + 158aa + 159bb + 160cc + 161dd + 162ee + 163ff + 164gg + 165hh + 166ii + 167jj + 168kk + 169ll + 170mm + 171nn + 172oo + 173pp + 174qq + 175rr + 176ss + 177tt + 178uu + 179vv + 180ww + 181xx + 182yy + 183zz + 184aa + 185bb + 186cc + 187dd + 188ee + 189ff + 190gg + 191hh + 192ii + 193jj + 194kk + 195ll + 196mm + 197nn + 198oo + 199pp + 200qq + 201rr + 202ss + 203tt + 204uu + 205vv + 206ww + 207xx + 208yy + 209zz + 210aa + 211bb + 212cc + 213dd + 214ee + 215ff + 216gg + 217hh + 218ii + 219jj + 220kk + 221ll + 222mm + 223nn + 224oo + 225pp + 226qq + 227rr + 228ss + 229tt + 230uu + 231vv + 232ww + 233xx + 234yy + 235zz + 236aa + 237bb + 238cc + 239dd + 240ee + 241ff + 242gg + 243hh + 244ii + 245jj + 246kk + 247ll + 248mm + 249nn + 250oo + 251pp + 252qq + 253rr + 254ss + 255tt + 256uu + 257vv + 258ww + 259xx + 260yy + 261zz + 262aa + 263bb + 264cc + 265dd + 266ee + 267ff + 268gg + 269hh + 270ii + 271jj + 272kk + 273ll + 274mm + 275nn + 276oo + 277pp + 278qq + 279rr + 280ss + 281tt + 282uu + 283vv + 284ww + 285xx + 286yy + 287zz + 288aa + 289bb + 290cc + 291dd + 292ee + 293ff + 294gg + 295hh + 296ii + 297jj + 298kk + 299ll + 300mm + 301nn + 302oo + 303pp + 304qq + 305rr + 306ss + 307tt + 308uu + 309vv + 310ww + 311xx + 312yy + 313zz + 314aa + 315bb + 316cc + 317dd + 318ee + 319ff + 320gg + 321hh + 322ii + 323jj + 324kk + 325ll + 326mm + 327nn + 328oo + 329pp + 330qq + 331rr + 332ss + 333tt + 334uu + 335vv + 336ww + 337xx + 338yy + 339zz + 340aa + 341bb + 342cc + 343dd + 344ee + 345ff + 346gg + 347hh + 348ii + 349jj + 350kk + 351ll + 352mm + 353nn + 354oo + 355pp + 356qq + 357rr + 358ss + 359tt + 360uu + 361vv + 362ww + 363xx + 364yy + 365zz + 366aa + 367bb + 368cc + 369dd + 370ee + 371ff + 372gg + 373hh + 374ii + 375jj + 376kk + 377ll + 378mm + 379nn + 380oo + 381pp + 382qq + 383rr + 384ss + 385tt + 386uu + 387vv + 388ww + 389xx + 390yy + 391zz + 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# TABLE OF LEARNING MATERIALS II-2 (Level M to X)

KUMON

SCT: Standard Completion Time (Min./Sheet)  
This is the set time in which the student should complete each worksheet, including time taken for correction.

March 2016																																									
M		SCT	N		SCT	O		SCT	XV		SCT	XM		SCT	XP		SCT																								
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## Level M (Cosine Rule)

**M184a**

For a triangle with sides  $a, b, c$  and angles  $A, B, C$  opposite to them respectively, the Cosine Rule states:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

When  $A = 90^\circ$ ,  $\cos A = 0$ , the Cosine Rule becomes the Pythagorean Theorem:  $a^2 = b^2 + c^2$ .

When  $A = 180^\circ$ ,  $\cos A = -1$ , the Cosine Rule becomes:  $a^2 = b^2 + c^2 + 2bc$ .

When  $A = 0^\circ$ ,  $\cos A = 1$ , the Cosine Rule becomes:  $a^2 = b^2 + c^2 - 2bc$ .

Example: In a triangle with sides  $a = 5$ ,  $b = 7$ , and  $c = 10$ , find the angle  $A$ .

Solution: Using the Cosine Rule,  $a^2 = b^2 + c^2 - 2bc \cos A$ .

$$5^2 = 7^2 + 10^2 - 2 \cdot 7 \cdot 10 \cos A$$

$$25 = 49 + 100 - 140 \cos A$$

$$25 = 149 - 140 \cos A$$

$$-124 = -140 \cos A$$

$$\cos A = \frac{124}{140} = \frac{31}{35}$$

$$A = \cos^{-1} \left( \frac{31}{35} \right) \approx 33.6^\circ$$

## Level N (Differentiation of Logarithmic Functions)

**N171a**

The derivative of  $\log_a x$  is:

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$$

For the natural logarithm,  $\ln x = \log_e x$ , the derivative is:

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

Example: Find the derivative of  $\log_2 x$ .

Solution:  $\frac{d}{dx} \log_2 x = \frac{1}{x \ln 2}$ .

Example: Find the derivative of  $\ln x^2$ .

Solution:  $\frac{d}{dx} \ln x^2 = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$ .

## Level O (Differentiation by Parts)

**O51a**

The Product Rule states:

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Example: Find the derivative of  $x \sin x$ .

Solution: Let  $u = x$  and  $v = \sin x$ .

$$\frac{d}{dx} (x \sin x) = x \cos x + \sin x$$

## Level XV (Vectors and Figures)

**XV99a**

1. Given that  $\vec{a} = 3\mathbf{i} + 4\mathbf{j}$  and  $\vec{b} = 5\mathbf{i} - 2\mathbf{j}$ , find the magnitude of  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ .

Solution:  $\vec{a} + \vec{b} = (3+5)\mathbf{i} + (4-2)\mathbf{j} = 8\mathbf{i} + 2\mathbf{j}$ .

$$|\vec{a} + \vec{b}| = \sqrt{8^2 + 2^2} = \sqrt{68} = 2\sqrt{17}$$

2. Given that  $\vec{a} = 3\mathbf{i} + 4\mathbf{j}$  and  $\vec{b} = 5\mathbf{i} - 2\mathbf{j}$ , find the angle between  $\vec{a}$  and  $\vec{b}$ .

Solution:  $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{(3)(5) + (4)(-2)}{\sqrt{3^2 + 4^2} \sqrt{5^2 + (-2)^2}} = \frac{15 - 8}{5 \sqrt{29}} = \frac{7}{5\sqrt{29}}$ .

$$\theta = \cos^{-1} \left( \frac{7}{5\sqrt{29}} \right) \approx 76.7^\circ$$

## Level XM (Transformations)

**XM62a**

1. Given that  $P(x, y)$  is transformed to  $P'(x', y')$  by the reflection in the  $y$ -axis, find the matrix representing this transformation.

Solution:  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ .

2. Given that  $P(x, y)$  is transformed to  $P'(x', y')$  by the reflection in the  $x$ -axis, find the matrix representing this transformation.

Solution:  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ .

## Level XP (Permutations and Combinations)

**XP24a**

1. Find the number of ways 4 people can sit around a round table.

Solution:  $\frac{4!}{4} = 3$  ways.

2. Find the number of ways 4 people can sit around a round table if one person is fixed.

Solution:  $3!$  ways.